APPLICATION OF THE LOGICS OF FUZZY SETS IN LAW ON THE EXAMPLE OF THE CASE OF “ASSETS COMING INDIRECTLY FROM A PROHIBITED ACT”

INTRODUCTORY REMARKS

The issue presented in this paper concerns money laundering (Art. 299 of the Criminal Code\(^2\)), which can be tackled most effectively and be prevented by a legal instrument and, most of all, by the confiscation of the assets “coming” (directly or indirectly) from a criminal offence according to Art. 44-45a of the Criminal Code and more precisely on the ground of Art. 299 § 7 of the Criminal Code – by confiscation of the assets coming

\(^1\) PhD; lecturer at the University of Rzeszów.
\(^2\) PhD, LL.M. (LMU Munich), lecturer at Wyższa Szkoła Handlowa in Kielce.
\(^1\) The designation “obtained” which appears first of all in Art. 45 § 2-3 of the Criminal Code and Art. 291-292 of the Criminal Code, has been deliberately omitted in this paper because it requires further research.
from the catalogue of primary offences. Cutting off the crime from the „dirty” property that comes, most commonly, from crime - is most effectively done by confiscating this property following Art. 44-45a of the Criminal Code and Art. 299 § 7 of the Criminal Code. This applies not only to the most serious forms of organized crime, but also to professional crime and to crime related to terrorism. Hence, it is important to consider the practical use of the mechanisms mentioned in the three articles of the Polish Criminal Code, emphasizing the weakest link of these regulations, which is discussed later in the paper.

In connection with the above, this paper attempts to solve the problem found in the Polish legal literature by M. Pregnel using the logic of fuzzy sets. This is about the designation of the „origin” of the property from the forbidden act, where this designation acts as a kind of link between the designation of „property” and the „forbidden act” mentioned in Art. 44-45 of the Criminal Code and Art. 299 § 7 of the Criminal Code. This is a problematic issue particularly on the ground of Art. 299 § 7 of the Criminal Code. While it is not fundamentally a problem to determine whether the property comes from a “directly” forbidden act (especially owing to a relatively transparent structure of the regulation Art. 44 of the Criminal Code), its “indirect” coming from this act is highly debatable (also on the ground of Art. 45 of the Criminal Code), which is displayed by insightful comments made by M. Prengel. For this reason it has become the subject of this article.

It is important here to point out the universality of the attempts made by M. Prengel on the question of the designation “property derives indirectly from a forbidden act”. Therefore the observations included in this paper refer respectively to the designation of “origin” in legal systems other than Polish, namely both to national law, i.e. in particular German, Austrian and Swiss, as well as the European Union and international law. In the above mentioned cases, one can successfully

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3 The notion of “primary offences” is introduced by e.g. T. Wróbel, Zakres tzw. źródłowych czynów zabronionych – przestępstwo prania brudnych pieniędzy w kontekście regulacji międzynarodowych, „Czasopismo Prawa Karnego i Nauk Penalnych” 2012, no. 4, p. 93-122.
4 See M. Prengel, Środki zwalczania przestępczości prania pieniędzy w ujęciu prawnoporównawczym (Eine Rechtsvergleichende Erfassung der Bekämpfungsmittel der Geldwäsche kriminalität), Toruń 2003, p. 325-332.
5 We assume that the designation “means of payment, financial instruments, securities, foreign exchange values, property rights or other movable or immovable property” is the same as the “property” (Art. 299 § 1 of the Criminal Code). It is worth emphasizing that the Polish legislator has not relinquished the category of primary forbidden acts, which includes such forbidden acts from which the property subject to money laundering comes.
6 See above all § 261 of the German criminal code of 15 May 1871 RGBl. 1871, p. 128 ff.
7 See in particular § 165 of the Austrian criminal code of 23 January 1974 BGBl. No. 60/1974.
8 See mainly Art. 305bis1 and Art. 305ter1 of the Swiss criminal code of 21 December 1937 SR 311.0.
10 See for example Art. 6 with reference to Art. 2 letter e of the convention of the United Nations against international organized crime, approved of by the General Assembly of the United Nations on 15 November 2000 Journal of Laws of 2005, no. 18, item 158.
FORMAL ISSUES

In the mathematical sense, a set understood classically is a primary concept, that means that it is impossible to define it by means of more simple concepts. However, one can state intuitively that a set is a system of elements similar to one another in a sense. At the same time, it is possible to decide unequivocally whether or not a given element belongs to a particular set. Specifying a set by means of the characteristic function we have: if the set is denoted by A, then its characteristic function $\mu$ for a given element $x$ is given value 1, when $x$ belongs to $A$ - value 0, when $x$ does not belong to $A$. Hence we formally obtain:

$$\mu(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A \end{cases}$$

The case becomes more complicated when we deal with expressions belonging to the natural language. Let us say $A = \{\text{a set of tall people}\}$. Assuming that people being less than 150 cm tall are not considered tall; while people 180 cm tall and more – on the contrary: in the first case we assign the 0 level of belonging to the $A$ set; in the other case the level 1 of belonging to this set. In addition it is obvious that people with the height between 150 cm and 180 cm can also be considered „tall” with the level of affiliation between the values 0 and 1 thus forming a fuzzy set.

DEFINITION 1: the $A$ set referred to in a given space as $X$ has the form of:

$$A = \{x_\mu(x) : x \in X\}$$

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11 See M. Prengel, Środki..., p. 325-332. In recent years, two particularly important works have been published, which re-examine the problem of “infecting” property derived from a forbidden act (i.e. the designation of “comes from”): N. Bischofberger, Zur Auslegung des Tatbestandsmerkmals „Herrühren” im Rahmen des Straftatbestandes § 261 StGB, Tectum-Verl., Marburg 2010, passim; J. Monßen, Geldwäsche: Die Organisierte Kriminalität und die Infizierungstheorie, Diplom.de, Hamburg 2012, passim.

12 The basis of the formal system of fuzzy sets was presented in the most complete and accessible way by its creator L.A. Zadeh. See, for example, this author, Fuzzy Logic = Computing with Words, „IEEE Transactions of Fuzzy Systems” 1996, no. 2, p. 103-111; this author, Note on Fuzzy Languages, “Information Sciences”, 1969 , no. 4, p. 421-434; this author, Fuzzy Algorithms, “Information and Control” 1968, no. 12, p. 94-102; this author, Fuzzy Sets, „Information and Control”, 1965, no. 8, p. 338-353. A complete list of this author’s most important publications can be found on <https://www2.eecs.berkeley.edu/Pubs/Faculty/zadeh.html> [accessed: 27.06.2017]. The application of the logic of fuzzy sets is particularly visible through an analysis of the research on artificial intelligence. The following papers are exceptionally instructive in this matter: M. Deb [and other], Application of Artificial Intelligence (AI) in Characterization of the Performance-Emission Profile of a Single Cylinder CI Engine Operating with Hydrogen in Dual Fuel Mode: An ANN Approach with Fuzzy-Logic Based Topology Optimization, „International Journal of Hydrogen Energy”, 2016, no. 32, p. 14330-14350; P.S. Sajja, Computer Aided Development of Fuzzy, Neural and Neuro-Fuzzy Systems, „International Journal of Engineering and Applied Computer Science” 2017, no. 1, p. 10-17.
where \( \mu : X \to [0, 1] \) is the function of affiliation of the fuzzy set. Value \( \mu_A(x) \) is named the degree of affiliation of the X element to the A set and is a number from the range \([0,1]\). Defining a fuzzy set is thus equivalent to specifying a set of elements whose affiliations are considered along with their degree of affiliation. In turn, the set X is called a space or universe, on which a fuzzy set is defined.

**DEFINITION 2:** The product of fuzzy sets is given by the formula:

\[
\mu \cap x = \min[\mu_A(x), \mu_B(x)].
\]

To determine the degree of belonging of element x to the common part, an operation called t-norm or triangular norm is introduced. The following conditions are then met:

If the norm is the function \( t : [0, 1] \cdot [0, 1] \to [0, 1] \), then:

1. The t function is not decreasing with respect to both arguments, i.e.:
   \[
   (a \leq b \land c \leq d) \Rightarrow [t(a, c) \leq t(b, d)];
   \]
2. The t function is alternate, therefore: \( t(a, b) = t(b, a) \);
3. The t function is combined, thus: \( t(t(a, b), c) = t(a, t(b, c)) \);
4. The t function fulfills the initial condition: \( t(a, 1) = a \).

**DEFINITION 3:** The total of fuzzy sets is determined as follows:

\[
\mu \cup (x) = \max[\mu_a(x), \mu_B(x)].
\]

In order to determine the degree of belonging of an element x to the sum of two fuzzy sets, an operation called s-norm is used, which is a function \( s : [0, 1] \cdot [0, 1] \to [0, 1] \) if the following conditions have been met:

1. The s function is not decreasing with respect to both arguments, i.e.:
   \[
   (a \leq b \land b \leq d) \Rightarrow [s(a, c) \leq (b, d)];
   \]
2. The s function is alternate, therefore: \( s(a, b) = s(b, a) \);
3. The s function is combined, thus: \( s(s(a, b), c) = s(a, s(b, c)) \);
4. The s function fulfills the initial condition: \( s(a, 0) = a \).

It can be concluded that t-norms and s-norms consist in determining the logical value of conjunction and alternative of sentences.

**DEFINITION 4:** The complement of the fuzzy set A is defined by the function of belonging \( \mu_A(x) = 1 - \mu_A(x) \). It is connected with the logical operator of negation, namely the function \( n : [0, 1] \to [0, 1] \) meeting the following conditions:
(1) \[ n(n(a)) = a \]
(2) \[ a \leq b \rightarrow n(a) \geq n(b) \]
(3) \[ n(0) = 1. \]

**DEFINITION 5:** The fuzzy sets A and B defined on the X universe are equal when \( \mu_A(x) = \mu_B(x) : \forall x \epsilon X. \)

**DEFINITION 6:** The core of the fuzzy set is a sharp set \( x \epsilon X, \) whose degree of belonging to the A set equals the unity. Therefore: .

**DEFINITION 7:** The amount of the A fuzzy set is the greatest degree of belonging to this set, i.e.: \( hgt(A) \exists (x \epsilon X) : A(x). \) The A set is normal, when \( hgt(A) = 1. \)

**DEFINITION 8:** A fuzzy implication marked \( a \rightarrow b \) is called the function consistent on edges with binary implication and preserving extensionality, and thus:
(1) \( 0 \rightarrow 0 = 0 \rightarrow 1 = 1 \rightarrow 1, 1 \rightarrow 0 = 0; \)
(2) \( a \rightarrow b = f(a, b). \)

**DEFINITION 9:** An empty fuzzy set \( \phi \) in the X space is one whose function of belonging is given as follows: \( \mu_\phi(x) = 0 : \forall x \epsilon x. \)

**DEFINITION 10:** The fuzzy rule of the *modus ponens* concluding is defined by the pattern:

- prerogative: x is \( A' \),
- implication: if x is A, then y is B,
- conclusion: therefore y is B',

where \( A, A' \subseteq X \land B, B' \subseteq Y \) are fuzzy sets, while x and y are the so called linguistic variables.

**DEFINITION 11:** The fuzzy rule of concluding *modus tollens* is specified by the pattern:

- prerogative: y is B',
- implication: if x is A, then y is B,
- conclusion: x is \( A' \),

where \( A, A' \subseteq X \land B, B' \subseteq Y \) are fuzzy sets, while x or y are the so called linguistic variables\(^\text{13}\).

\(^\text{13}\) In order to fathom the basic knowledge of the logic of fuzzy sets it is necessary to understand fully the classic logics. More on this A. Grzegorczyk, *Zarys logiki matematycznej*, Warsaw 1984, *passim*; G. Hunter,
CASE STUDY

Let us assume that the x linguistic variable is the expression “property coming indirectly from a forbidden act”\(^\text{14}\). Let us also assume that it can have the following values: ”property not indirectly coming from a prohibited act”, ”property to a small extent coming indirectly from a prohibited act”, “property coming »a little« indirectly from a prohibited act”, “property half coming indirectly from a prohibited act”, “property to a large extent coming indirectly from a prohibited act”, “property in major part coming indirectly from a prohibited act”, “property completely coming indirectly from a prohibited act”\(^\text{15}\).

Thus the following degrees of belonging are obtained:

1. Property not indirectly coming from a prohibited act = 0;
2. Property to a small extent coming indirectly from a prohibited act = 0.2;
3. Property coming »a little« indirectly from a prohibited act = 0.4;
4. Property half coming indirectly from a prohibited act = 0.5;
5. Property to a large extent coming indirectly from a prohibited act = 0.6;
6. Property in major part coming indirectly from a prohibited act = 0.8;
7. Property completely coming indirectly from a prohibited act = 1.

It can be represented graphically as follows:

\[ \text{Property not indirectly coming from a prohibited act} = \text{np; Property completely coming indirectly from a prohibited act} = \text{pc; Property indirectly coming from a prohibited act} = \text{p}. \]


\(^{14}\) We can say that if the property does not come indirectly from a prohibited act, it also does not come directly from this act.

\(^{15}\) Despite the fact that this article is limited only to exemplarily selected seven values, an unlimited number of them can be easily presented.
Suppose, therefore, that John Kowalski purchased a car for PLN 50,000, of which PLN 500 came from the property of criminal origin. Asking if this car comes from this property, it turns out that:

1. In the case of the concept of “contamination” of the whole property 
\[ \mu_A(x) = 1; \]

2. If it is assumed that the “clean” part of the property launders the “dirty” one 
\[ \mu_A(x) = 0; \]

3. If it is considered that the property can be “contaminated” by percentage 
\[ 0 < \mu_A(x) < 1; \]

4. If it is assumed that the idea of quota “contamination” of property is in force 
\[ 0 \leq \mu_A(x) \leq 1. \]

Let us also assume that \( B = \{ \text{the set of prohibited acts contained in Art. 299 § 1 of the Criminal Code understood as a catalogue of primary prohibited acts} \} \). Let us also assume that the linguistic variable contained in set \( B \) assumes the following values: “prohibited act is not included in the acts referred to in Art. 299 § 1 of the Criminal Code”, “there is little doubt that the prohibited act is the act referred to in Art. 299 § 1 of the Criminal Code”, “there is little doubt that the prohibited act is the act referred to in Art. 299 § 1 of the Criminal Code”, “there is little doubt that the prohibited act is the act referred to in Art. 299 § 1 of the Criminal Code”, “there is little doubt that the prohibited act is the act referred to in Art. 299 § 1 of the Criminal Code”, “there is little doubt that the prohibited act is the act referred to in Art. 299 § 1 of the Criminal Code”, “there is little doubt that the prohibited act is the act referred to in Art. 299 § 1 of the Criminal Code”, “there is little doubt that the prohibited act is the act referred to in Art. 299 § 1 of the Criminal Code”. Thus we have the following degrees of belonging:

1. “prohibited act is not included in the acts referred to in Art. 299 § 1 of the Criminal Code” = 0;
2. “there is little doubt that the prohibited act is the act referred to in Art. 299 § 1 of the Criminal Code” = 0.25;
3. “there are little-justified doubts whether the prohibited act is the act referred to in Art. 299 § 1 of the Criminal Code” = 0.38;
4. “there are medium-justified doubts as to whether the prohibited act is the act referred to in Art. 299 § 1 of the Criminal Code” = 0.5;
5. “the evidence to a large extent indicates that the prohibited act is the act referred to in Art. 299 § 1 of the Criminal Code” = 0.5.

\(^{16}\) It was based on M. Prengel’s reflections, *Środki zwalczania…*, p. 328-332.
referred to in Art. 299 § 1 of the Criminal Code” = 0.75;

(6) “the evidence strongly points out that the prohibited act is the act referred to in Art. 299 § 1 of the Criminal Code” = 0.9;

(7) “there is no doubt that the prohibited act is the act referred to in Art. 299 § 1 of the Criminal Code” = 1.

The graphical representation of the degrees of belonging of the X linguistic variable contained in set B is analogous to the graph for set A. Comparing the degrees of belonging of the linguistic variable for sets A and B we obtain respectively:

(1) \[ A(x) = [x_1 = 0, x_2 = 0.2, x_3 = 0.4, x_4 = 0.5, x_5 = 0.6, x_6 = 0.8, x_7 = 1] \]
(2) \[ B(x) = [x_1 = 0, x_2 = 0.25, x_3 = 0.38, x_4 = 0.5, x_5 = 0.75, x_6 = 0.9, x_7 = 1] \]

It is easy to see that both the core \((A) = \{\text{property wholly deriving from the prohibited act}\} = 1\) and the core \((B) = \{\text{there is no doubt that the prohibited act is the act referred to in Art. 299 § 1 of the Criminal Code}\} = 1\). In addition: \(\text{hgt}(A) = \{\text{property completely deriving indirectly from the forbidden act}\} = 1\) and \(\text{hgt}(B) = \{\text{there is no doubt that the prohibited act is the act referred to in art. 299 § 1 of the Criminal Code}\} = 1\). It can be concluded that sets A and B are normal. The following dependencies also exist:

(1) \[ A \cup B = [x_1 = 0, x_2 = 0.25, x_3 = 0.4, x_4 = 0.5, x_5 = 0.75, x_6 = 0.9, x_7 = 1] \]
(2) \[ A \cap B = [x_1 = 0, x_2 = 0.2, x_3 = 0.38, x_4 = 0.5, x_5 = 0.6, x_6 = 0.8, x_7 = 1] \]

Let us now apply the assumptive rule of concluding modus tollens in the form: \(B' = A' \cdot (A \rightarrow B)\). We know that Łukasiewicz’s implications are in the form of \((A \rightarrow B) = 1 \land (1 - A \lor B)\). In the case of the analysed sets A and B it is true that:

\((A \rightarrow B) = [x_1 = 0, x_2 = 0.2, x_3 = 0.38, x_4 = 0.5, x_5 = 0.25, x_6 = 0.1, x_7 = 0] \)

Considering the t-norm minimum given by the equatio:

\((A \rightarrow B)_T = (A \land B)\) we ultimately obtain the final dependency:

\[ B' = A' \cdot (A \rightarrow B) = [x_1 = 0, x_2 = 0.2, x_3 = 0.38, x_4 = 0.5, x_5 = 0.4, x_6 = 0.2, x_7 = 0] \]

The values obtained unequivocally indicate that the completion of the set \(B = \{\text{prohibited acts are not included in those acts, which have been specified in Art. 299 § 1 of the Criminal Code}\}\) occurs when the values of the linguistic variable x directly indicate that the property even less than “a little” comes indirectly from the forbidden act\(^{17}\).

\(^{17}\) We neglect the value of the variable x5 because the analysed set is not normal.
CONCLUSIONS

The article shows how strict formal logic tools can be used to analyse the meaning of at least some of the legal language concepts, although it seems possible in the legal language as well. By using the elementary instrumental logic of fuzzy sets it has been shown that the meaning of the expression “property indirectly coming from the forbidden act” cannot be analysed in binary terms. The obtained result gives the reason for a cautious postulate that the meaning of other terms of the language of the law and legal language can be also described by means of the formalism of the logic of fuzzy sets. This is also a contribution to the discussion on the precision and explicitness of the terms used in the language of the law and legal language.

Bibliography


See pioneer papers by L. Philipps, in which attempts were undertaken to apply the logic of fuzzy sets in some criminal law institutions. Further by this author, Unbestimmte Rechtsbegriffe und Fuzzy Logic, [in:] Strafgerechtigkeit: Festschrift für Arthur Kaufmann zum 70. Geburtstag, ed. F. Haft [and others], Müller, Heidelberg 1993, p. 265-289; as above, Kompensatorische Verknüpfungen in der Rechtsanwendung – ein Fall für Fuzzy Logic, [in:] Festschrift für Günther Jahr zum siebzigsten Geburtstag: Vestigia iuris, ed. M. Martinek and others), Mohr, Tübingen 1993, p. 169-180; as above, Ein bißchen Fuzzy Logic für Juristen, [in:] Institutionen und Einzelne im Zeitalter der Informationstechnik: Machtpositionen und Rechte, ed. M.-T. Tinnefeld [and others], Oldenbourg, München [and others]: 1994, p. 219-224. It seems possible to apply the logic of fuzzy sets in other unclear (“fuzzy”) terms of criminal law such as for example: intention and inadvertence. One might suspect that the application of the logics of fuzzy sets combined with the modal sentence logic could bring results in relation to the complex institution of the law like for example: judicial dimension of punishment. The determination of it, however, requires further research. The syllogistic model of applying the law and its relation to the solutions of the logic of fuzzy sets was commented on by L. Philipps, G. Sartor, Introduction: from Legal Theories to Neural Networks and Fuzzy Reasoning, “Artificial Intelligence and Law” 1999, no. 7, p. 115-128.


Summary: In this article an attempt was made to formulate a fundamental distinction for the consideration of criminal law, namely “property coming indirectly from a prohibited act”. For this purpose, fuzzy set logic tools were used. Finally, it has been shown that it is possible to apply strict logical concepts to the language of the law and legal language.

Keywords: logic of fuzzy sets, criminal law, combatting organised crime, “dirty” property, recovery of the crime property

**ZASTOSOWANIE LOGIK ZBIORÓW ROZMYTYCH W PRAWIE: NA PRZYKŁADZIE ZNAMENIA „MIENIE POCHODZĄCE POŚREDNIO Z CZYNU ZABRONIONEGO”**

Streszczenie: W artykule podjęto próbę formalizacji znamienia fundamentalnego do rozważań z zakresu prawa karnego, a mianowicie – „mienie pochodzące pośrednio z czynu zabronionego”. W tym celu wykorzystano narzędzia logiki zbiorów rozmytych. Ostatecznie wykazano, że jest możliwe stosowanie ścisłych koncepcji logicznych do języka prawnego i prawniczego.

Słowa kluczowe: logika zbiorów rozmytych, prawo karne, zwalczanie przestępczości zorganizowanej, „brudne” mienie, odebranie mienia przestępczości